A Truly Concurrent Semantics for the $\mathbb{K}$ Framework Based on Graph Transformations

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The $\mathbb{K}$ Framework

`http://k-framework.org`

What is K?

A tool-supported rewrite-based framework for defining programming language design and semantics.

Why?

- Programming languages must have formal semantics!
- And analysis/verification tools should build on them
  - Otherwise they are adhoc and likely wrong

The 𝕂 Framework

Defining programming languages

- Java 1.4 (Chen, CAV'06)
- Scheme (Hills&Meredith, SCHEME'07)
- Verilog (Meredith&Katelman, MEMOCODE'10)
- C (Chucky Ellison, POPL'12)
- In progress: Haskell, LLVM, Javascript, ...
- Paradigmatic teaching languages (functional, object-oriented, agents-based)

The $\mathbb{K}$ Framework

Tool support

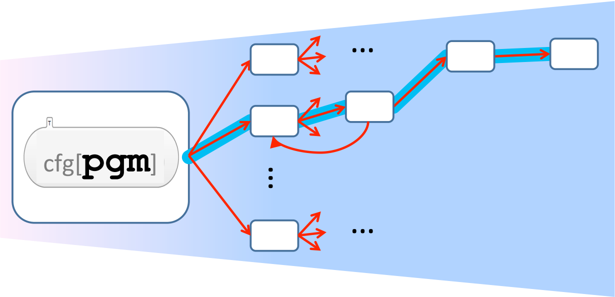
- Efficient and interactive execution (interpreters)
- State-space exploration (search and model-checking)
- Deductive program verification (in progress)

Leveraging the generic tool support given by the Maude rewrite engine

The 𝕂 Framework

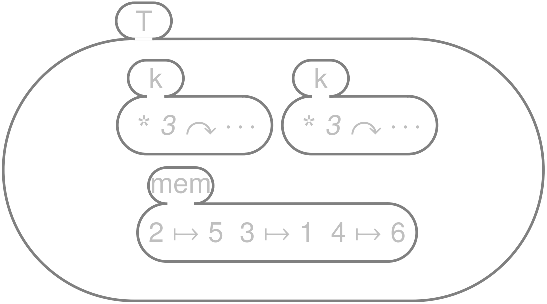
Rewriting based

- Running configurations represented as first order terms
- Rules specify allowed transitions between configurations
- Semantics as a transition system



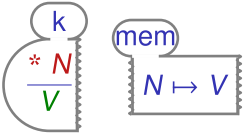
𝕂 configurations and rules

Running configuration

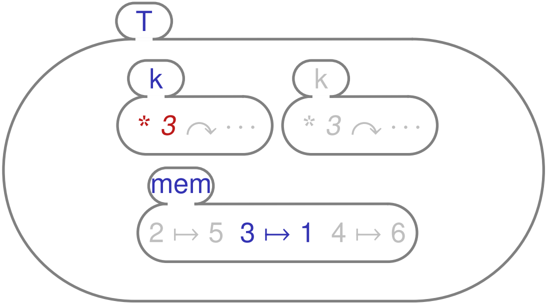


𝕂 configurations and rules

Read rule



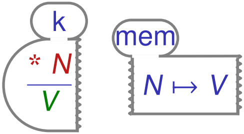
Running configuration



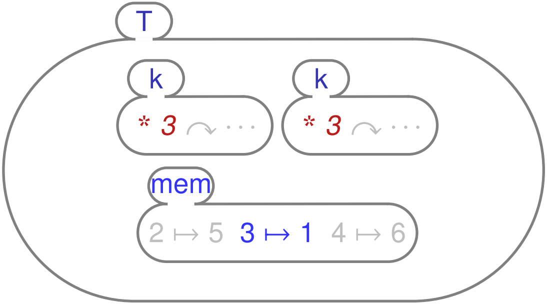
𝕂 configurations and rules

More concurrency with 𝕂 rules?

Read rule



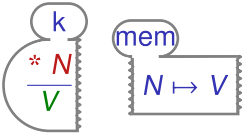
Running configuration



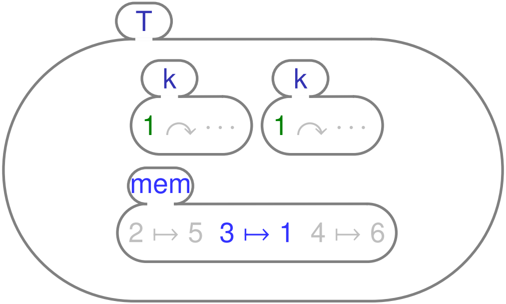
$\mathbb{K}$ configurations and rules

More concurrency with $\mathbb{K}$ rules!

Read rule



Running configuration



Semantics Requirements

- Conservative extension of term rewriting
- While allowing as much concurrency as possible

$$(1)\ h(\underline{\ \ x\ \ }_{g(x,x)}, y, \underline{1}_{0}) \qquad (2)\ h(x, \underline{0}_{1}, y) \qquad (3)\ \underline{a}_{b} \qquad (4)\ \underline{f(x)}_{x}$$

$$h(f(a),0,1) \xRightarrow{(1)+(2)+(3)+(4)} h(g(b,b),1,0)$$

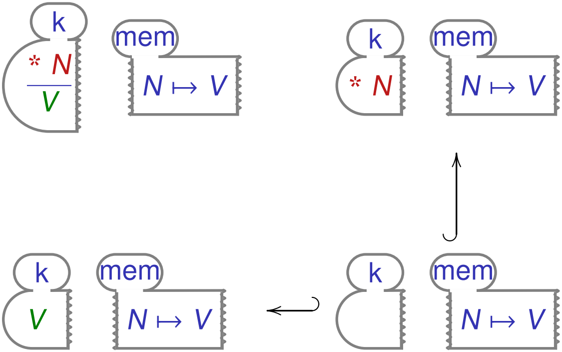
$\mathbb{K}$ rules resemble graph transformation rules

The DPO approach

Graph transformation rule (DPO)

$$K \cup L \hookleftarrow K \hookrightarrow K \cup R$$

$\mathbb{K}$ rule



Concurrency and serializability in graph transformations

$\rho_1 : L_1 \hookleftarrow K_1 \hookrightarrow R_1$ $\rho_2 : L_2 \hookleftarrow K_2 \hookrightarrow R_2$

$$\rho_1 \coprod \rho_2$$

$$\begin{tikzcd}
L_1 \coprod L_2 \arrow[d, "m"] & K_1 \coprod K_2 \arrow[l, hook'] \arrow[r, hook] \arrow[d, dashed] & R_1 \coprod R_2 \arrow[d, dashed] \\
G & C \arrow[l, hook', dashed] \arrow[r, hook, dashed] & H
\end{tikzcd}$$

Theorem (Parallelism and serializability [*Ehrig, Kreowski, 1976*]**)**

If $G \overset{m,\rho_1}{\Longrightarrow} H_1$, and $G \overset{m,\rho_2}{\Longrightarrow} H_2$ are parallel independent,i.e., only overlapping on the read-only part, then (1) $G \xrightarrow{m,\rho_1 \coprod \rho_2} H$ (concurrency); and (2) $H_1 \overset{\rho_2}{\Longrightarrow} H$ and $H_2 \overset{\rho_1}{\Longrightarrow} H$ (serializability).

Formally capturing the concurrency of $\mathbb{K}$

Given that ...

- $\mathbb{K}$ rules resemble graph transformation rules
- Graph rewriting captures concurrency with sharing of context

Capture the concurrency intended for $\mathbb{K}$ through graph rewriting

- Term graph rewriting approaches seem a promising start
  - Are sound and complete w.r.t. term rewriting
  - Are special forms of graph transformations
- However, term graph rewriting have 0-sharing (like term rewriting)

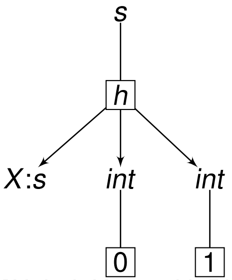
Representing terms as graphs

Term

$$h(X, 0, 1)$$

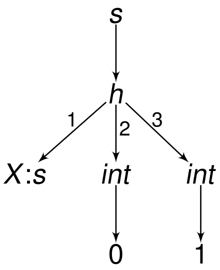
where X is a variable, h and X are of sort s, and 0 and 1 are integers

Jungle (hypergraph) representation



[*Habel, Kreowski, Plump, 1987*]

Graph representation



𝕂 graph rules: a new kind of term graph rewriting rules

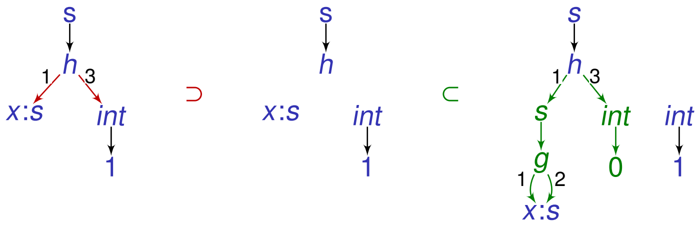
𝕂 rule ρ

$$h(\underset{g(x,x)}{\underline{\quad x \quad}}, y, \underset{0}{\underline{1}})$$

Direct representation as a rewrite rule $K2R(\rho)$

$$h(x, y, 1) \rightarrow h(g(x, x), y, 0)$$

Corresponding graph rewrite rule $K2G(\rho)$



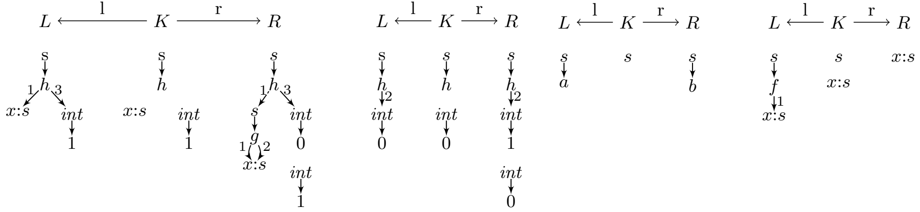
Desired level of parallelism

(1): $h(\frac{x}{g(x,x)}, y, \frac{1}{0})$

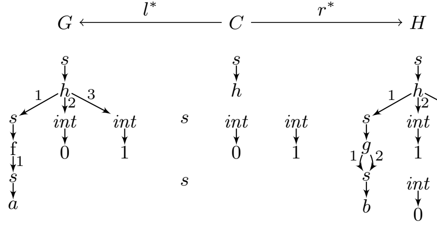
(2): $h(x, \frac{0}{1}, y)$

(3): $\frac{a}{b}$

(4): $\frac{f(x)}{x}$



$$h(f(a),0,1) \xRightarrow{(1)+(2)+(3)+(4)} h(g(b,b),1,0)$$



$\mathbb{K}$ rewriting

Definition

Let $\mathbb{S}$ be a $\mathbb{K}$ rewrite system and t be a term. Then

$$t \overset{\mathbb{S}}{\underset{\mathbb{K}}{\Longrightarrow}} t' \quad \text{iff} \quad K2G(t) \xrightarrow[Graph]{K2G(\mathbb{S})} H \text{ such that } \mathrm{term}(H) = t'$$

Theorem (Correctness w.r.t. rewriting)

Soundness: *If* $t \overset{\rho}{\underset{\mathbb{K}}{\Longrightarrow}} t'$ *then* $t \xrightarrow[Rew]{K2R(\rho)} t'$.

Completeness: *If* $t \xrightarrow[Rew]{K2R(\rho)} t'$ *then* $t \overset{\rho}{\underset{\mathbb{K}}{\Longrightarrow}} t'$.

Serializability: *If* $t \overset{\rho_1+\cdots+\rho_n}{\underset{\mathbb{K}}{\Longrightarrow}} t'$ *then* $t \overset{\rho_1{}^*}{\underset{\mathbb{K}}{\Longrightarrow}} \cdots \overset{\rho_n{}^*}{\underset{\mathbb{K}}{\Longrightarrow}} t'$ *(and thus,* $t \xrightarrow[Rew]{*} t'$*).*

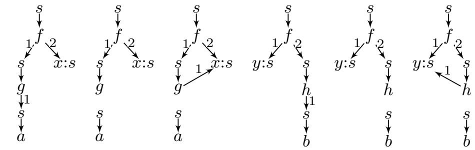
Instead of proof

- $\mathbb{K}$ term graphs are stable under concurrent applications of $\mathbb{K}$ rules
  - If their instances only overlap on read-only part
  - If they do not introduce cycles
- Serializability based on graph rewriting serializability
  [*Ehrig, Kreowski, 1976*]
- $\mathbb{K}$ graph rewriting conservatively extends Jungle rewriting
  - For terms without subterm sharing
- Jungle rewriting is sound and complete w.r.t. rewriting
  [*Holland, Plump, 1991; Plump 1999*]

Parallel 𝕂 graph rewriting can introduce cycles

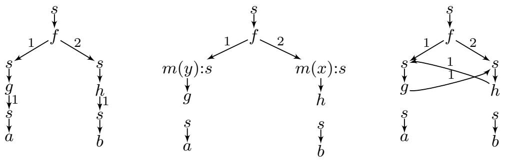
$$(1):\ f(g(\underset{x}{\underline{a}}), x) \qquad\qquad (2):\ f(y, h(\underset{y}{\underline{b}}))$$

$$L \xleftarrow{\ l\ } K \xrightarrow{\ r\ } R \qquad\qquad L \xleftarrow{\ l\ } K \xrightarrow{\ r\ } R$$



$$f(g(a), h(b)) \xRightarrow{(1)+(2)} \text{infinite term } f(g(h(g(h(\ldots)))), h(g(h(g(\ldots)))))$$

$$G \xleftarrow{\ l^*\ } C \xrightarrow{\ r^*\ } H$$

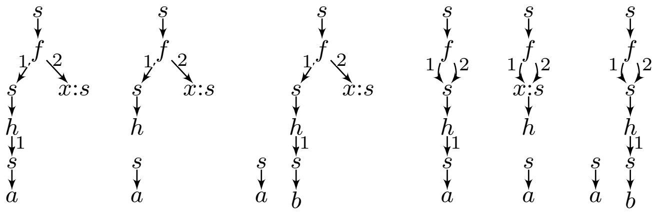


Subterm sharing can jeopardize soundness

$$(1):\ f(h(\underset{b}{\underline{a}}), x) \qquad\qquad f(h(a), h(a)) \overset{(1)}{\Rrightarrow} f(h(b), h(b))$$

(unsound rewriting)

$$L \xleftarrow{\quad l \quad} K \xrightarrow{\qquad r \qquad} R \qquad\qquad G \xleftarrow{l^*} C \xrightarrow{r^*} H$$



Conclusions

Results: A new formalism of term-graph rewriting

- Sound and complete w.r.t. term rewriting
- Capturing the intended concurrency of $\mathbb{K}$ rewriting

Future work

- Investigate the cycle condition
- Special graph representations for lists and multisets
  - And re-prove the correctness for this representation
- Tools which take advantage of this new semantics?