CityHash: Fast Hash Functions for Strings

Geoff Pike
(joint work with Jyrki Alakuijala)

Google

http://code.google.com/p/cityhash/

Introduction

- Who?
- What?
- When?
- Where?
- Why?

Outline

Recent activity

- SHA-3 winner was announced last month
- Spooky version 2 was released last month
- MurmurHash3 was finalized last year
- CityHash version 1.1 will be released this month

In my backup slides you can find ...

- My notation
- Discussion of cyclic redundancy checks
  - What is a CRC?
  - What does the `crc32q` instruction do?

Traditional String Hashing

- Hash function loops over the input
- While looping, the *internal state* is kept in registers
- In each iteration, consume a fixed amount of input

Traditional String Hashing

- Hash function loops over the input
- While looping, the *internal state* is kept in registers
- In each iteration, consume a fixed amount of input
- Sample loop for a traditional byte-at-a-time hash:

```
for (int i = 0; i < N; i++) {
    state = Combine(state, B_i)
    state = Mix(state)
}
```

Two more concrete old examples (loop only)

```
for (int i = 0; i < N; i++)
    state = ρ_{-5}(state) ⊕ B_i
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```

$$\texttt{for (int } i = 0;\ i < N;\ i\texttt{++)}$$
$$\quad state = \rho_{-5}(state) \oplus B_i$$

```
for (int i = 0; i < N; i++)
    state = 33 · state + B_i
```

$$\texttt{for (int } i = 0;\ i < N;\ i\texttt{++)}$$
$$\quad state = 33 \cdot state + B_i$$

A complete byte-at-a-time example

```
// Bob Jenkins circa 1996
int state = 0
for (int i = 0; i < N; i++) {
   state = state + B_i
   state = state + σ_{-10}(state)
   state = state ⊕ σ_6(state)
}
state = state + σ_{-3}(state)
state = state ⊕ σ_{11}(state)
state = state + σ_{-15}(state)
return state
```

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What's better about this? What's worse?

What Came Next—Hardware Trends

- CPUs generally got better
  - Unaligned loads work well: read words, not bytes
  - More registers
  - SIMD instructions
  - CRC instructions
- Parallelism became more important
  - Pipelines
  - Instruction-level parallelism (ILP)
  - Thread-level parallelism

What Came Next—Hash Function Trends

- People got pickier about hash functions
  - Collisions may be more costly
  - Hash functions in libraries should be “decent”
  - More acceptance of complexity
  - More emphasis on *diffusion*

Jenkins' *mix*

Also around 1996, Bob Jenkins published a hash function with a 96-bit input and a 96-bit output. Pseudocode with 32-bit registers:

$$
\begin{array}{lll}
a = a - b; & a = a - c; & a = a \oplus \sigma_{13}(c) \\
b = b - c; & b = b - a; & b = b \oplus \sigma_{-8}(a) \\
c = c - a; & c = c - b; & c = c \oplus \sigma_{13}(b) \\
a = a - b; & a = a - c; & a = a \oplus \sigma_{12}(c) \\
b = b - c; & b = b - a; & b = b \oplus \sigma_{-16}(a) \\
c = c - a; & c = c - b; & c = c \oplus \sigma_{5}(b) \\
a = a - b; & a = a - c; & a = a \oplus \sigma_{3}(c) \\
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c = c - a; & c = c - b; & c = c \oplus \sigma_{5}(b) \\
a = a - b; & a = a - c; & a = a \oplus \sigma_{3}(c) \\
b = b - c; & b = b - a; & b = b \oplus \sigma_{-10}(a) \\
c = c - a; & c = c - b; & c = c \oplus \sigma_{15}(b)
\end{array}
$$

Thorough, but pretty fast!

Jenkins' *mix*-based string hash

Given *mix(a, b, c)* as defined on the previous slide, pseudocode for string hash:

```
uint32 a = ...
uint32 b = ...
uint32 c = ...
int iters = ⌊N/12⌋
for (int i = 0; i < iters; i++) {
    a = a + W_{3i}
    b = b + W_{3i+1}
    c = c + W_{3i+2}
    mix(a, b, c)
}
```

etc.

Modernizing Google's string hashing practices

- Until recently, most string hashing at Google used Jenkins' techniques
  - Some in the "32-bit" style
  - Some in the "64-bit" style, whose *mix* is 4/3 times as long
- We saw Austin Appleby's 64-bit Murmur2 was faster and considered switching

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- We saw Austin Appleby's 64-bit Murmur2 was faster and considered switching
- Launched education campaign around 2009
  - Explain the options; give recommendations
  - Encourage labelling: "may change" or "won't"

Quality targets for string hashing

There are roughly four levels of quality one might seek:

- quick and dirty
- suitable for a library
- suitable for *fingerprinting*
- secure

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Is Murmur2 good for a library? for fingerprinting? both?

Murmur2 preliminaries

First define two subroutines:

$$\textit{ShiftMix(a)} = a \oplus \sigma_{47}(a)$$

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and

$$\textit{TailBytes(N)} = \sum_{i=1}^{N \bmod 8} 256^{(N \bmod 8)-i} \cdot B_{N-i}$$

Murmur2

```
uint64 k = 14313749767032793493
int iters = ⌊N/8⌋
uint64 hash = seed ⊕ Nk
for (int i = 0; i < iters; i++)
    hash = (hash ⊕ (ShiftMix(W_i·k)·k))·k

if (N mod 8 > 0)
    hash = (hash ⊕ TailBytes(N))·k

return ShiftMix(ShiftMix(hash) · k)
```

Murmur2 Strong Points

- Simple
- Fast (assuming multiplication is fairly cheap)
- Quality is quite good

Questions about Murmur2 (or any other choice)

- Could its speed be better?
- Could its quality be better?

Murmur2 Analysis

Inner loop is:

```
for (int i = 0; i < iters; i++)
    hash = (hash ⊕ f(W_i)) · k
```

$$\texttt{hash} = (\texttt{hash} \oplus f(W_i)) \cdot k$$

where *f* is “Mul-ShiftMix-Mul”

Murmur2 Speed

- ILP comes mostly from parallel application of f
- Cost of *TailBytes(N)* can be painful for $N < 60$ or so

Murmur2 Quality

- f is invertible
- During the loop, diffusion isn't perfect

Testing

Common tests include:

- Hash a bunch of words or phrases
- Hash other real-world data sets
- Hash all strings with edit distance <= *d* from some string
- Hash other synthetic data sets
  - E.g., 100-word strings where each word is “cat” or “hat”
  - E.g., any of the above with e x t r a s p a c e
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  - E.g., any of the above with e x t r a s p a c e
- We use our own plus *SMHasher*
- *avalanche*

Avalanche (by example)

Suppose we have a function that inputs and outputs 32 bits. Find M random input values. Hash each input value with and without its j^{th} bit flipped. How often do the results differ in their k^{th} output bit?

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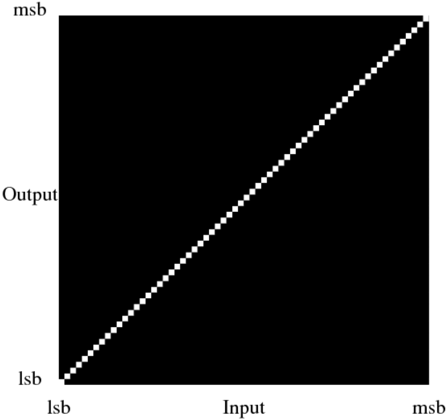
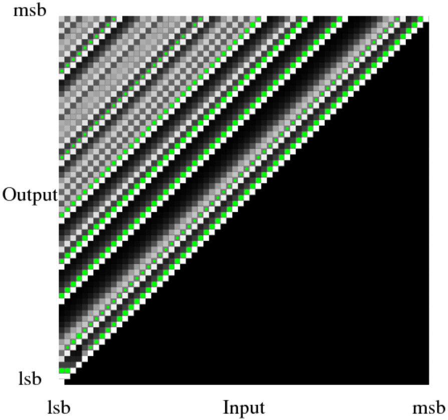
Ideally we want “coin flip” behavior, so the relevant distribution has mean $M/2$ and variance $1/4M$.

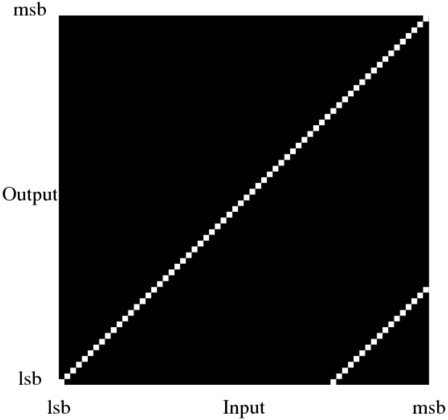
64x64 avalanche diagram: $f(x) = x$



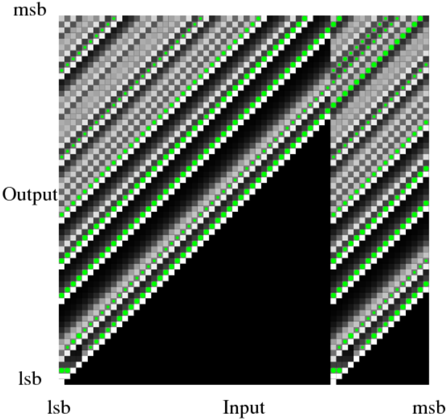
64x64 avalanche diagram: $f(x) = kx$



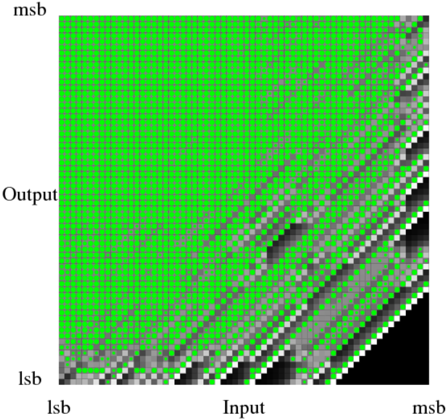
64x64 avalanche diagram: *ShiftMix*



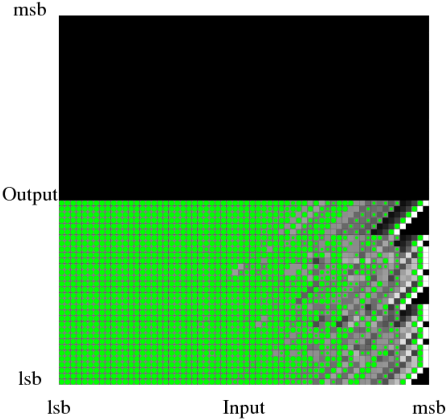
64x64 avalanche diagram: *ShiftMix(x)* $\cdot$ *k*



64x64 avalanche diagram: *ShiftMix(kx)* $\cdot k$



64x64 avalanche diagram: $f(x) = CRC(kx)$



The CityHash Project

Goals:

- Speed (on Google datacenter hardware or similar)
- Quality
  - Excellent diffusion
  - Excellent behavior on all contributed test data
  - Excellent behavior on basic synthetic test data
  - Good internal state diffusion—but not too good, cf. Rogaway's Bucket Hashing

Portability

For speed without total loss of portability, assume:

- 64-bit registers
- pipelined and superscalar
- fairly cheap multiplication
- cheap $+, -, \oplus, \sigma, \rho, \beta$
- cheap register-to-register moves

Portability

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- cheap $+, -, \oplus, \sigma, \rho, \beta$
- cheap register-to-register moves
- $a + b$ may be cheaper than $a \oplus b$
- $a + cb + 1$ may be fairly cheap for $c \in \{0, 1, 2, 4, 8\}$

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How many dynamic branches are reasonable for hashing a 12-byte input?

How many arithmetic operations?

CityHash64 initial design (2010)

- Focus on short strings
- Perhaps just use Murmur2 on long strings
- Use overlapping unaligned reads
- Write the minimum number of loops: 1
- Focus on speed first; fix quality later

The CityHash64 function: overall structure

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```
if (N <= 32)
   if (N <= 16)
     if (N <= 8)
       ...
     else
       ...
   else
     ...
else if (N <= 64) {
  // Handle 33 <= N <= 64
  ...
} else {
   // Handle N > 64
   int iters = ⌊N/64⌋
   ...
}
```

The CityHash64 function (2012): preliminaries

Define $\alpha(u, v, m)$:

$$
\begin{array}{llll}
\text{let} & a & = & u \oplus v \\
 & a' & = & \mathit{ShiftMix}(a \cdot m) \\
 & a'' & = & a' \oplus v \\
 & a''' & = & \mathit{ShiftMix}(a'' \cdot m) \\
\text{in} & & & \\
 & a''' \cdot m & &
\end{array}
$$

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\text{in} & & & \\
 & a''' \cdot m & &
\end{array}
$$

Also, k_0, k_1, and k_2 are primes near 2^{64}, and K is $k_2 + 2N$.

CityHash64: *1 <= N <= 3*

$$
\begin{aligned}
\text{let} \quad a &= B_0 \\
b &= B_{\lfloor N/2 \rfloor} \\
c &= B_{N-1} \\
y &= a + 256b \\
z &= N + 4c
\end{aligned}
$$

in

$$\mathit{ShiftMix}((y \cdot k_2) \oplus (z \cdot k_0))$$

CityHash64: *4 <= N <= 8*

$$\alpha(N + 4W_0^{32}, W_{-1}^{32}, K)$$

CityHash64: *9 <= N <= 16*

$$
\begin{array}{llll}
\text{let} & a & = & W_0 + k_2 \\
& b & = & W_{-1} \\
& c & = & \rho_{37}(b) \cdot K + a \\
& d & = & (\rho_{25}(a) + b) \cdot K \\
\text{in} & & & \\
& \alpha(c, d, K) & &
\end{array}
$$

CityHash64: *17 <= N <= 32*

$$\begin{array}{llll} \text{let} & a & = & W_0 \cdot k_1 \\ & b & = & W_1 \\ & c & = & W_{-1} \cdot K \\ & d & = & W_{-2} \cdot k_2 \\ \text{in} & & & \end{array}$$

$$\alpha(\rho_{43}(a + b) + \rho_{30}(c) + d, a + \rho_{18}(b + k_2) + c, K)$$

CityHash64: *33 <= N <= 64*

$$
\begin{array}{llcl}
\text{let} & a & = & W_0 \cdot k_2 \\
 & e & = & W_2 \cdot k_2 \\
 & f & = & W_3 \cdot 9 \\
 & h & = & W_{-2} \cdot K \\
 & u & = & \rho_{43}(a + W_{-1}) + 9(\rho_{30}(W_1) + c) \\
 & v & = & a + W_{-1} + f + 1 \\
 & w & = & h + \beta((u + v) \cdot K) \\
 & x & = & \rho_{42}(e + f) + W_{-3} + \beta(W_{-4}) \\
 & y & = & (\beta((v + w) \cdot K) + W_{-1}) \cdot K \\
 & z & = & e + f + W_{-3} \\
 & r & = & \beta((x + z) \cdot K + y) + W_1 \\
 & t & = & \mathit{ShiftMix}((r + z) \cdot \mathrm{K} + \mathrm{W}_{-4} + h) \\
\text{in} & & & \\
 & tK + x & &
\end{array}
$$

Evaluation for *N* <= 64

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- CityHash64 is about 1.5x faster than Murmur2 for N <= 64
- Quality meets targets (bug reports are welcome)
- Simplifying it would be nice

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- CityHash64 is about 1.5x faster than Murmur2 for N <= 64
- Quality meets targets (bug reports are welcome)
- Simplifying it would be nice
- Key lesson: Don’t loop over bytes
- Key lesson: Understand the basics of machine architecture
- Key lesson: Know when to stop

Next steps

Arguably we should have written CityHash32 next. That's still not done.

Instead, we worked on 64-bit hashes for $N > 64$, and 128-bit hashes.

CityHash64 for $N > 64$

The one loop in CityHash64:

- 56 bytes of state
- 64 bytes consumed per iteration
- 7 rotates, 4 multiplies, 1 xor, about 36 adds (??)
- influenced by *mix* and Murmur2

128-bit CityHash variants

- CityHash128
  - same loop body, manually unrolled
  - slightly faster for large N
- CityHashCrc128
  - totally different function
  - uses CRC instruction, but isn't a CRC
  - faster still for large N

Evaluation for $N > 64$

Evaluation for $N > 64$

- CityHash64 is about 1.3 to 1.6x faster than Murmur2
- For long strings, the fastest CityHash variant is about 2x faster than the fastest Murmur variant
- Quality meets targets (bug reports are welcome)
- Jenkins' Spooky is a strong competitor

My recommendations

For hash tables or fingerprints:

	Nehalem, Westmere, Sandy Bridge, etc.	similar CPUs	other CPUs
small *N*	CityHash	CityHash	TBD
large *N*	CityHash	Spooky or CityHash	TBD

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	Nehalem, Westmere, Sandy Bridge, etc.	similar CPUs	other CPUs
small N	CityHash	CityHash	TBD
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For quick-and-dirty hashing: Start with the above

Future work

- CityHash32
- Big Endian
- SIMD

The End

The End

Backup Slides

Notation

$$
\begin{aligned}
N &= \text{the length of the input (bytes)} \\
a \oplus b &= \text{bitwise exclusive-or} \\
a + b &= \text{sum (usually mod } 2^{64}\text{)} \\
a \cdot b &= \text{product (usually mod } 2^{64}\text{)} \\
\sigma_n(a) &= \text{right shift } a \text{ by } n \text{ bits} \\
\sigma_{-n}(a) &= \text{left shift } a \text{ by } n \text{ bits} \\
\rho_n(a) &= \text{right rotate } a \text{ by } n \text{ bits} \\
\rho_{-n}(a) &= \text{left rotate } a \text{ by } n \text{ bits} \\
\beta(a) &= \text{byteswap } a
\end{aligned}
$$

More Notation

$$
\begin{aligned}
B_i &= \text{the } i^{th} \text{ byte of the input (counts from 0)} \\
W_i^b &= \text{the } i^{th}\ b\text{-bit word of the input}
\end{aligned}
$$

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B_i &= \text{the } i^{th} \text{ byte of the input (counts from 0)} \\
W_i^b &= \text{the } i^{th}\ b\text{-bit word of the input} \\
W^b_{-1} &= \text{the last } b\text{-bit word of the input} \\
W^b_{-2} &= \text{the second-to-last } b\text{-bit word of the input}
\end{aligned}
$$

Cyclic Redundancy Check (CRC)

The commonest explanation of a CRC is in terms of polynomials whose coefficients are elements of GF(2).

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The commonest explanation of a CRC is in terms of polynomials whose coefficients are elements of GF(2). In GF(2):

0 is the additive identity,
1 is the multiplicative identity, and
$1 + 1 = 0 + 0 = 0$.

CRC, part 2

Sample polynomial:

$$p = x^{32} + x^{27} + 1$$

CRC, part 3

We can use p to define an equivalence relation: We'll say q and r are equivalent iff they differ by a polynomial times p.

CRC, part 4

Theorem: The equivalence relation has $2^{\mathrm{Degree}(p)}$ elements.

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Lemma: if $\text{Degree}(p) = \text{Degree}(q) > 0$
then $\text{Degree}(p + q) < \text{Degree}(p)$
and, if not, $\text{Degree}(p + q) = \max(\text{Degree}(p), \text{Degree}(q))$

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and, if not, $\text{Degree}(p + q) = \max(\text{Degree}(p), \text{Degree}(q))$

Observation: There are $2^{\text{Degree}(p)}$ polynomials with degree less than $\text{Degree}(p)$, none equivalent.

CRC, part 5

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$\text{Degree}(x^{50}) - \text{Degree}(p) = 18$; therefore $x^{50} - x^{18} \cdot p$ has degree less than 50.

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Example: What is a degree <= 31 polynomial equivalent to x^{50}?

Degree(x^{50}) $-$ Degree(p) $= 18$; therefore $x^{50} - x^{18} \cdot p$ has degree less than 50.

$$
\begin{aligned}
x^{50} - x^{18} \cdot p &= x^{50} - x^{18} \cdot (x^{32} + x^{27} + 1) \\
&= x^{50} - (x^{50} + x^{45} + x^{18}) \\
&= x^{45} + x^{18}
\end{aligned}
$$

CRC, part 6

Applying the same idea repeatedly will lead us to the lowest degree polynomial that is equivalent to x^{50}.

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The result:

$$x^{50} \equiv x^{30} + x^{18} + x^{13} + x^{8} + x^{3}$$

CRC, part 7

More samples:

$$
\begin{aligned}
x^{50} &\equiv x^{30} + x^{18} + x^{13} + x^{8} + x^{3} \\
x^{50} + 1 &\equiv x^{30} + x^{18} + x^{13} + x^{8} + x^{3} + 1 \\
x^{51} &\equiv x^{31} + x^{19} + x^{14} + x^{9} + x^{4} \\
x^{51} + x^{50} &\equiv x^{31} + x^{30} + x^{19} + x^{18} + x^{14} + x^{13} + x^{9} + x^{8} + x^{4} + x^{3} \\
x^{51} + x^{31} &\equiv x^{19} + x^{14} + x^{9} + x^{4}
\end{aligned}
$$

CRC in Practice

- There are thousands of CRC implementations
- We'll focus on those that use `_mm_crc32_u64()` or `crc32q`
- The inputs are a 32-bit number and a 64-bit number
- The output is a 32-bit number

What is `crc32q`?

`crc32q` for inputs u and v returns
$C(u \text{ xor } v) = F(E(D(u \text{ xor } v)))$.

$D(0) = 0, D(1) = x^{95}, D(2) = x^{94}, D(3) = x^{95} + x^{94}, D(4) = x^{93}, \ldots$

E maps a polynomial to the equivalent with lowest-degree.

$F(0) = 0, F(x^{31}) = 1, F(x^{30}) = 2, F(x^{31} + x^{30}) = 3, F(x^{29}) = 4, \ldots$

How is `crc32q` used?

C operates on 64 bits of input, so:

For a 64-bit input, use $C(\text{seed}, u_0)$.

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For a 128-bit input, use $C(C(\text{seed}, u_0), u_1)$.

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C operates on 64 bits of input, so:

For a 64-bit input, use $C(\text{seed}, u_0)$.

For a 128-bit input, use $C(C(\text{seed}, u_0), u_1)$.

For a 192-bit input, use $C(C(C(\text{seed}, u_0), u_1), u_2)$.

C as matrix-vector multiplication

A 32×64 matrix times a 64×1 vector yields a 32×1 result.

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The matrix and vectors contain elements of GF(2):